

Note on the Stability of Jacobi's Ellipsoid.

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It is known that Maclaurin's spheroid of rotating liquid becomes unstable when its eccentricity reaches the value $\sin 54^\circ 21' 27''$. This is a form of bifurcation, and for increasing momentum the stability passes over to Jacobi's ellipsoid. It is possible to prove these results by the method applied by me to the discussion of the stability of the pear-shaped figure of equilibrium,* and it is worth while to do so, because we obtain thereby a verification of the complicated analysis used in the previous investigation, and because the series which arise are exactly similar to the former series.

In vol. 3 of my 'Scientific Papers' I shall give a few details about the present analysis. It will here suffice to say that it gives the known results correctly, and a good approximation to the form of Jacobi's ellipsoid.

M. Liapounoff maintains that if, in considering the pear-shaped figure, I had taken more terms into account, the conclusion arrived at would have been reversed. It is therefore worth while to examine the corresponding series in the present precisely similar problem, in which a few terms are found to give good results. Although, of course, we cannot thus prove the correctness of my former conclusion, as against M. Liapounoff's contention, yet I think that this investigation tends to indicate that I was right before.

In the case of Jacobi's ellipsoid there are two series proceeding *pari passu*, viz., those depending on the zonal and on the fourth tesseral harmonics. The reader is requested to refer to my former papers for the interpretation of the notation. For these the results are as follows:—

<i>Zonal Harmonics.</i>			<i>Fourth Tesseral Harmonics.</i>		
<i>i.</i>	$(B_i)^2/C_i + [i, 0].$	Ratio of each term to the next.	<i>i.</i>	$(B_i^4)^2/C_i^4 + [i, 4].$	Ratio of each term to the next.
2	37·2393	37	4	3·3307	10
4	1·0077	$6\frac{1}{2}$	6	0·3352	13
6	0·1554	9	8	0·0253	—
8	0·0170	—			

In the case of the pear-shaped figure we need only consider the zonal terms, since the tesseral terms are clearly insignificant. The result was

* 'Phil. Trans.,' A, vol. 200 (1902), pp. 251—314, and vol. 208 (1908), pp. 1—19.

i .	$(B_i)^2/C_i + [i, 0]$.	Ratio of each term to the next.
2	-0·000080868	—
4	0·000247274	3·9
6	0·000062350	3·2
8	0·000019576	3·1
10	0·000006328	—

We may conjecture that the uncomputed residue is about 0·000003.* The convergence is clearly slower than in the case of Jacobi's ellipsoid, but I cannot think that the series can converge so slowly as to justify M. Liapounoff.

It seems to me, then, that this present investigation should lead us to look with doubt on an argument adverse to the former conclusion.

On the Wave-lengths of Lines in the Secondary Spectrum of Hydrogen.

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The spectrum of hydrogen is one which has engaged the attention of numerous observers from the year 1851 onwards. Historically it is of great interest, firstly owing to the part it played in the controversy between Ångström and Wüllner on the plurality of spectra, and secondly, because it was the first spectrum in which a relation between the wave-lengths of the lines composing it was discovered.

It is not proposed to discuss the origin of the two hydrogen spectra in this communication, as it has recently been done in a very complete manner by Dufour;† the main points at issue, however, may perhaps be briefly stated.

Wüllner,‡ who was the first to make detailed observations of the spectra of hydrogen under different conditions, described three spectra. The first consisted of three bright lines, the second of a number of bands, and the third of several bright lines mostly in the green. This was disputed by

* The absolute magnitudes of the numbers in the series compared are very different, but this merely depends on the definitions chosen for the several harmonic functions.

† 'Ann. Chim. Phys.' (8), vol. 9, pp. 361—432, 1906.

‡ 'Phil. Mag.' (4), vol. 37, p. 405, 1869, and vol. 39, p. 365, 1870.